الموضوع: ________ تبولجي (Topology) التاريخ: / /

Definition:- let X be a non empty set
let $T \subseteq P(X)$

$P(X) = \{ A : A \subseteq X \}$. بور الاكس:- هو مجموعة كل المجاميع الجزئية من X.

T is topology on X iff:

1) $T \subseteq P(X)$
2) $X, \emptyset \in T$
3) if $A_1, A_2, \dots, A_n \in T$ Then $A_1 \cap A_2 \cap \dots \cap A_n \in T$
4) if $\{A_i\}_{i \in \alpha}$ is a callection of element in T then $\bigcup_{i \in \alpha} A_i \in T$.

Ex:- let $X = \mathbb{R}$
$T = \{ (a,b) : a, b \in \mathbb{R} \}$
$(X = \mathbb{R}; T)$ is called Topology space.

Ex:- Is (X, T) T.S. such that $X = \{1, 2, 3\}$

$T = \{ X, \emptyset, \{1\}, \{2\} \}$

Sol:- No, it is not T.S.

since: $\{1\} \in T$
$\{2\} \in T$
but $\{1, 2\} \notin T$.

تبولجي:- هو علم يهتم بدراسة الاشكال وتشابه بينهم
وبدراسة الاشكال من ناحية مطاطية.

1

Ex:- $X = \{1, 2, 3, 4\}$

$T = \{X, \phi, \{1\}, \{2\}, \{1,2\}\}$ is (X, T) T.s.

(Topological space. فضاء تبولجي

Sol:- 1) $X, \phi \in T$

2) التقاطع يحقق لان
نقاطع كل ينتمي ب T

3) الاتحاد يحقق لان اتحاد كل مجاميع
so, (X, T) is T.S.

Ex:- $X = \{1, 2, 3, 4\}$

$T = \{X, \phi, \{1\}, \{3\}, \{1,2\}\}$ is (X, T) T.S.

Solution:- (X, T) is not T.S.

$\{1\} \in T$

$\{3\} \in T$

but $\{1\} \cup \{3\} = \{1, 3\} \notin T$.

Ex:- let $X = A = \{1, 2, \ldots, 100\}$

let $T \{B \subseteq A : 1 \in B\}$

هنا مثال ليس تبولجي لان لازم كل مجاميع بيها 1 و
ϕ لازم مابيها 1 $\therefore$ ϕ لاتنتمي ال T وتكون ~~...~~ ليس
تبولجي

Ex: let $X = A = \{1, 2, \cdots, 100\}$

let $T = \{B \subseteq A : 1 \in B\} \cup \{\emptyset\}$

is (X, T) T.S. ? Why?

sol:- ① $\emptyset \in T$

2) $1 \in X \Rightarrow X \in T$

3) let $A_1 \in T$, $A_2 \in T$ T.P $A_1 \cap A_2 \overset{?}{\in} T$

since: $A_1 \in T \Rightarrow 1 \in A_1$ * اذا عنصر موجود بالمجموعه

$A_2 \in T \Rightarrow 1 \in A_2$ الاولى وموجود بالمجموعه ثانيه

So, $1 \in A_1 \cap A_2 \Rightarrow A_1 \cap A_2 \in T$ معناها عنصر موجود بالتقاطع

دليل انديكس indexst set

4) $\forall A_i$, $i \in \alpha$, $A_i \in T$

i.e. $1 \in A_i, \forall i \Rightarrow 1 \in \bigcup_{i \in \alpha} A_i \Rightarrow \bigcup A_i \in T$

يعني الاتحاد ينتمي ب T

$\therefore (X, T)$ is T.S.

3

The interior of set:- داخلية

let (X, T) T.S. The interior of a set $A \subseteq X$ denoted by (يرمز) $(int(A))$ and it is the Union (اتحاد) of all open subset of A. (اتحاد كل مجموعة مفتوحة)

* كل عناصر تبولجي تعتبر مجاميع مفتوحة.

* مجموعه مفتوحه هي الي عناصرها نقاط داخليه في مجموعه.

Remark:- $A \in T \Leftrightarrow A$ is open

Ex:- $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$T = \{A, \emptyset, \{1\}, \{2\}, \{1, 2\}\}$

find: $int(A)$, $int(\emptyset)$, $int(\{1\})$, $int(\{1,3\})$, $int(\{5,6\})$.

sol:- $int(A) = A$

A

$int(\emptyset) = \emptyset$

$int(\{1\}) = \{1\}$

$int(\{1,3\}) = \{1\}$

$int(\{5,6\}) = \emptyset$

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Remark :- let (X, T) T.s.

$A \subseteq X$, A is closed $\Leftrightarrow$ A^c is open.

(co finite topology)

H.w:- let $A = \{1, 2, 3, \ldots\}$

$T = \{B \subseteq A : B^c \text{ is finite}\} \cup \{\emptyset\}$

show That (A, T) T.s.

$A^c = \emptyset$

عدد كاردينلتي ل $\emptyset = 0$

و 0 عدد منتهي ∴ $\emptyset$ تكون finite

sol:- $\emptyset \in T$

$A^c = \emptyset$ finite

$\therefore A \in T$

الموضوع: ________ التاريخ: / /

The clasuer of set :

let (X,T) T.s. $A \subseteq X$ so The clasuer of A denoted by $cL(A)$ is The intersection $(\cap)$ of all closed مغلقة set containing A.

Ex:- $A = \{1,2,3,4,5,6,7,8,9,10\}$

$T = \{A, \emptyset, \{1\}, \{1,2\}, \{2\}\}$ find $cL(A)$, $cL(\emptyset)$, $cL(\{1\})$, $cL(\{1,3\})$, $cL(\{5,6\})$.

$T^c = \{\emptyset, A, \{2,\dots,10\}, \{1\} \cup \{3,\dots,10\}, \{3,\dots,10\}\}$.

Sol:- $cL(A) = A$

$cL(\emptyset) = \emptyset$

$cL(\{1\}) = \{1\} \cup \{3,\dots,10\}$

$cL(\{2\}) = \{2,\dots,10\}$

$cL(\{1,3\}) = \{3,\dots,10\} \cup \{1\}$

$cL(\{5,6\}) = \{3,\dots,10\}$

Ex:- $X = \{a,b,c\}$

هذا مثال يعلمنه انو المجموعه تكدر نطلع منها اكثر من تبولجي واحد

$T_1 = \{X, \emptyset, \{a\}\}$, $T_2 = \{X, \emptyset, \{b\}\}$ Topology on X

is $T_1 \cap T_2$ Topology on X

is $T_1 \cup T_2$ Topology on X

معناها هو اصغر تبولجي يحوي فقط X و Ø

sol:- $T_1 \cap T_2 = \{X, \emptyset\}$ indiscrete topolgy

$T_1 \cup T_2 = \{X, \emptyset, \{a\}, \{b\}\}$ is not topolgy since

$\{a\} \in T_1 \cup T_2$, $\{b\} \in T_1 \cup T_2$ لان اتحاد $\{a\},\{b\}$ لا يوجد

but $\{a\} \cup \{b\} = \{a,b\} \notin T_1 \cup T_2$

الموضوع: التاريخ: / /

انبت ان تقاطع 2 تبولوجي على X ايضا تبولوجي

Q/ show That The intersetion of two topologies on X is also topology.

Proof:- let T_1 Top. on X

T_2 Top. on X

1) $X, \emptyset \in T_1$

$X, \emptyset \in T_2$

$X, \emptyset \in T_1 \cap T_2$

2) let $A \in T_1 \cap T_2$

$B \in T_1 \cap T_2$

T.P. $B \cap A \overset{?}{\in} T_1 \cap T_2$

$\because A \in T_1 \cap T_2 \Rightarrow A \in T_1$, $A \in T_2$

$\because B \in T_1 \cap T_2 \Rightarrow B \in T_1$, $B \in T_2$

$\therefore A \in T_1$, $B \in T_2$

$\therefore A \cap B \in T_1$

also $A \cap B \in T_2$

$\therefore A \cap B \in T_1 \cap T_2$

3) (H.W) اتحاد واجب

limit point of a set :- let (X, T) T.s.

نقاط هي

مجموعه $A \subseteq X$ ودائره الي هي حوالها هي G

$P \in X$

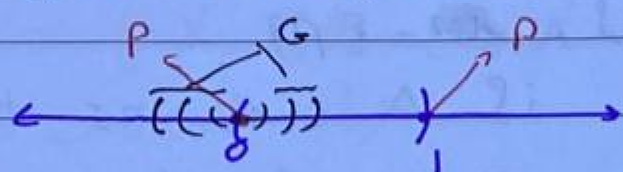
متغير P is limit point of a set A if satisfied The following condition :- اذا حققت شرط الاتي

اكلمت P هي نقطة غاية للمجموعة A

If $G \in T$ (G is open set) and $P \in G$.

Then : $(G - \{P\}) \cap A \neq \emptyset$

ملاحظ Ex :-



0 limite point of $(0,1)$?

$(G - \{0\}) \cap (0,1) \neq \emptyset$

Ex :- let $X = \{a, b, c, d, e\}$

$T = \{X, \emptyset, \{a\}, \{c,d\}, \{a,c,d\}, \{b,c,d,e\}\}$

let $B = \{a, b, c\}$

Is b limit point of B ?

Sol :- The open sets That containing b are :-

X and $\{b, c, d, e\}$

$(X - \{b\}) \cap B = \{a, c\} \neq \emptyset$

$(\{b,c,d,e\} - \{b\}) \cap B = \{c\} \neq \emptyset$

$\therefore$ b is limint point of B.

Remark: The set of all limit points of a set A is called derived set of A and denoted by A'.

مجموعه كل نقاط الـ limit مجموعه تسمى A

Ex: let X be any set $T = \{X, \emptyset\}$

$A \subseteq X$, describe The derived set of A.

او وصف

Sol: $A' = \begin{cases} \emptyset & \text{if } A = \emptyset \\ X - \{P\} & \text{if } A = \{P\} \\ X & \text{if } A \text{ contains two or more points} \end{cases}$

* دائماً X و ∅ يكون مجموعات مغلقة ومفتوحة

Ex: let X be a set

$T = T_c$ = co-finite topology on X

$T_c = \{A \subseteq X : A^c \text{ is finite}\}$

let $A \subseteq X$, X is infinite set describe then $cL(A)$?

(H.w.) $\boxed{CL(A) = A \text{ in } A \text{ is finite}}$

Sol: let A is finite

so, A^c is infinite $\Rightarrow A^c \in T_c$

A^c is open

$(A^c)^c = A$ is closed

$CL(A) = A$

Remark: if A is closed then $cL(A) = A$

* اذا مجموعه مغلقة فدائماً كلوجر يطلع مجموعه نفسها

$cL(A^c) = X$ (H.w.) if A is infinite then $cL(A) = X$

الموضوع:

التاريخ: / /

Remark:- $CL(A) = A \cup A'$ تعريف جديد للكلوجر

Definition:- let (X, T) T.S. $A \subseteq X$, A is dense iff $CL(A) = X$.
كثافة

Ex:- $X = \{1, 2, 3, \ldots\}$

$T = \{X, \phi, \{1\}\}$

is $\{1\}$ dense? why?

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Sol:- The only closed set containting $\{1\}$ is
مجموعة مغلقة وحيدة تحوي

$\therefore CL(\{1\}) = X$

$\therefore \{1\}$ dense.

Ex:- $(\mathbb{R}, T_u)$

T_u هو تبولوجي معرف على مجموعة اعداد حقيقية عناصرها فترات مفتوحة.

$CL(Q) = \mathbb{R}$

since, $\forall a \in \mathbb{R}$

a is limit Point of Q

$CL(Q) = Q \cup Q'$

$= Q \cup \mathbb{R}$

$= \mathbb{R}$

$\therefore Q$ is dense set in $(\mathbb{R}, T_u)$.

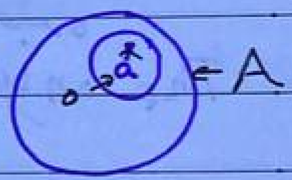
الموضوع: ________ التاريخ: / /

Ex1: $(\mathbb{R}, T_u)$

$$Cl((0,1)) = (0,1) \cup (0,1)'$$
$$= (0,1) \cup [0,1].$$
$$= [0,1]$$

*اكول عن نقطه هي انتيرير المجموعه اذا لكينه مجموعه مفتوحه تحويها وتكون جزئيه من مجموعه

Def:- let (X,T) T.S. $A \subseteq X$, $a \in A$. a is interior point of A if: $\exists\, O \in T$ s.t.
$a \in O \subseteq A$.



Def:- $int(A)$ is the set of all interior points of A.

Ex:- $A = \{1, 2, \dots, 10\}$
$T = \{A, \emptyset, \{1\}, \{2\}, \{1,2\}\}$
$B = \{5, 6\}$ find $int(B)$

Solution:- $5 \in B$, 5 is not inter point of B, since $\nexists$ ~~open~~ $O \in T$ s.t. $5 \in O \subseteq B$

الموضوع: 　　　　　　　　　　　　　　　　　التاريخ: / /

خارج

Definition:- let (X,T) T.S., $A \subseteq X$, The exterior of a set A is the interior of the set A^c.

Ex:- let $X=\{a,b,c\}$
T topology defined on X.
by $T=\{X, \emptyset, \{a\}\}$
find $ext(\{a,c\})$

sol:- let $A=\{a,c\} \Rightarrow A^c=\{b\}$

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$int(\{b\}) = \emptyset = ext(\{a,c\})$

تحتوي بداخله
مجموعة

Definition:- let (X,T) T.S.

let $A \subseteq X$ the boundary of a set A is denoted by $b(A)$. And its the set of all elements dose not belong to interior A or exterier A.

i.e. $b(A) = \left\{ a \in X : \begin{array}{l} a \notin int(A) \\ a \notin ext(A) \end{array} \right\}$

Ex:- let $X=\{a,b,c\}$
$T=\{X, \emptyset, \{a\}\}$
$A=\{a,c\}$
find $b(A) = ?$

Solution:- 1) $ext(A) = \emptyset$

2) $int(A) = \{a\}$

$\therefore b(A) = \{b, c\}$

Also find $cL(A) = ?$

$T^c = \{\emptyset, X, X - \{a\}\}$ أو $T^c = \{\emptyset, X, \{b, c\}\}$

$X \in T^c$, $A \subseteq X$

$cL(A) = X$

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Def:- let (X, T) T.s.

$A \subseteq X$, A is called nowhere dence iff:

$int(cL(A)) = \emptyset$

Ex:- let $X = \{a, b, c\}$

$T = \{X, \emptyset, \{a\}\}$ find nowhere dence.

sol:- $T^c = \{\emptyset, X, \{b, c\}\}$

$cL(A) = X$

$int(X) = \emptyset$ $\quad X$

$\therefore (X, T)$ is nowhere dence.

Definition:- let (X, T) T.s. , $N \subseteq X$

جوار

$P \in X$, N is called neighborhood of a point p iff:

$\exists\, G \in T$ s.t. $p \in G \subseteq N$.

التاريخ: / / الموضوع: ________

Ex: let $X = \{a, b, c\}$

$T = \{X, \emptyset, \{a\}\}$. $b \in X$, $\underset{N}{\{a, b\}} \subseteq X$. is $\{a, b\}$ neighberhood of a point b ?

Sol: $\nexists\, G \in T$ s.t. $b \in G \subseteq \{a, b\}$

$\therefore$ $N = \{a, b\}$ is not neighberhood of b .

* كل مجموعة مفتوحة هي جوار لكن العكس غير صحيح.

Ex: let $(X, T) = (\mathbb{R}, T_u)$

is $[-1, 1]$ neighberhood a point 0 ?

Sol: yes , since

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$0 \in (-1, 1) \subseteq [-1, 1]$

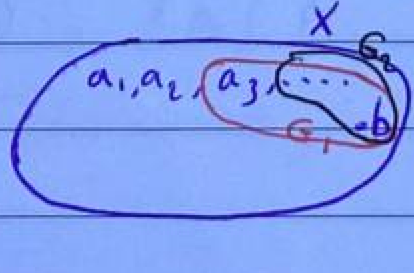
$(-1, 1)$ is open set

Definition: let (X, T) T.S. , A sequence (متتابعة) of points $\langle a_1, a_2, \dots \rangle$ in X is called converges (تقترب) to a point $b \in X$ (denoted by $\lim a_n = b$) iff :

$\forall$ open set G s.t. $b \in G$ we have:

$\exists$ a positive integer $n_0 \in \mathbb{N}$

such that : $\forall\, n > n_0$ we have $a_n \in G$.



Ex:- let (X, T_{in}) be let indiscrete top~~o~~ological space.
بسيطة

i.e, $T_{in} = \{X, \emptyset\}$. يكون تبولوجي

let $a_n = \langle a_1, a_2, \dots \rangle$ be a seqaence of points in X
let $b \in X$
is $\lim a_n = b$.

تحوي
Sol:- since the only open set containing b is X.

and X contains all the terms of a_n, so
$\lim a_n = b$
تقترب
س/ اعطي مثال على متتابعة تحوي على اكثر من نقطة.

Ex:- let (X, T_d) be the discrete topology let
تكون كل مجاميع جزئية من X تكون مفتوحة تنتمي بـ تبولوجي

$a_n = \langle a_1, a_2, a_3, \dots \rangle$ be a sequence in X let $b \in X$.
is $\lim \overset{?}{=} b$
$a_n \to b$

Sol:- since $\{b\}$ is open set (i.e, $\{b\} \in T_d$)
and $\{b\}$ dose not contain any element of the sequence a_n
$\therefore \lim a_n \neq b$

الموضوع: ______

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but in This example we have the sequence

$a_b = \langle a_1, a_2, \dots, a_n, b, b, b, b, \dots \rangle$

$\lim a_b = b$

Remark:- let (X, T) T.S. and T_1, T_2 are topologies on X and $T_1 \subseteq T_2$

T_2 is finer of T_1.

or T_1 is coarser of T_2.

مقارنة

Ex: $X = \{a, b, c\}$

$T_2 = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$

$T_1 = \{X, \emptyset, \{a\}\}$

$T_1 \subseteq T_2$

Definition:- let (X, T) T.S. $Y \subseteq X$, then T_Y is topology on Y as a subspace topology of (X, T) iff

هذا (او مجموعة)

$T_Y = \{A : A = Y \cap O, \ O \in T\}$ 15

Example:- let (X, T) T.S. such that $X = \{a, b, c, d\}$

$T = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$

let $Y = \{a, c\}$, find T_Y or The subspace topology on Y.

Sol:- $T_Y = \{X \cap Y, \emptyset \cap Y, \{a\} \cap Y, \{b\} \cap Y, \{a, b\} \cap Y\}$

$= \{Y, \emptyset, \{a\}\}$

مو بالضرورة المجموعه المفتوحه ب T_Y تكون نفسها ب T_1

Ex: let $(\mathbb{R}, T_u)$ be the usual topology.

مثال مهم

Is every open set in T_y is open set in T? هذا السؤال

جواب اكتب مثال و حله

Sol: let. $Y = [3, 8]$

Then $\underbrace{(2, 5)}_{\in T_u} \cap \underbrace{[3, 8]}_{\in Y} \in T_y$ 16

since $(2, 5)$ is open set in T_u

So, $(2, 5) \cap [3, 8] \in T_y$

and $(2, 5) \cap [3, 8] = [3, 5) \in T_y$

So, $[3, 5)$ is open set in the subspace topology in T_y.

and $[3, 5) \notin T_u$ ← لأنه ما موجودة بالمفتوحة

الموضوع: التاريخ: / /

موجود بالكتاب

P.73:- Solved problems:-

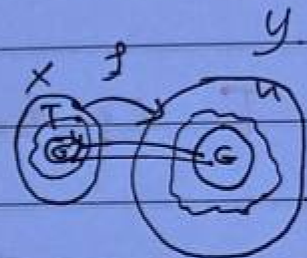
1) H.W مطلوب بس هو موجود بالكتاب

u هو تبولوجي على y

5) let $f : X \longrightarrow (Y, \tilde{u})$

let $T = \{ f^{-1}(G) : G \in u \}$ show that T is topology on X.

Solution:-



1) $f^{-1}(Y) = X \in T$

2) $f^{-1}(\emptyset) = \emptyset \in T$

2) let $A_1, A_2 \in T$

T.P $A_1 \cap A_2 \overset{?}{\in} T$

since $A_1, A_2 \in T \therefore \exists\, G_1, G_2 \in u$

such that: $f^{-1}(G_1) = A_1$, $f^{-1}(G_2) = A_2$

$A_1 \cap A_2 = f^{-1}(G_1) \cap f^{-1}(G_2)$

$= f^{-1}(G_1 \cap G_2)$

since $G_1 \cap G_2 \in u$

$\therefore f^{-1}(G_1 \cap G_2) \in T$

$\therefore A_1 \cap A_2 \in T$.

3) ~~let $A_1, A_2 \in T$~~ let A_i, $i \in I$ be a class of

~~T.P $A_1 \cap A_2 \overset{?}{\in} T$~~ open sets in T.

$\therefore \exists\, G_i$, $i \in I$ open set in u

such that $A_i = f^{-1}(G_i)$, $i \in I$

$\therefore \bigcup A_i = \bigcup f^{-1}(G_i)$

$= f^{-1}(\bigcup G_i)$

since $G_i \in \mathcal{U} \ \forall i \in I$

$\mathcal{U}$ is topology on Y

$\therefore \bigcup G_i \in \mathcal{U}$

$\therefore f^{-1}(\bigcup G_i) \in T$

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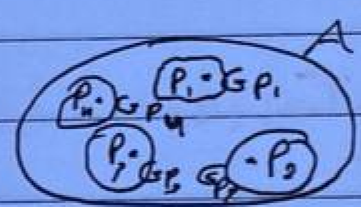
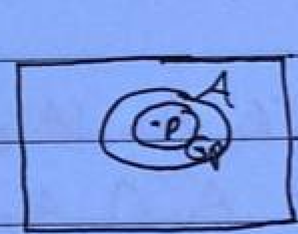
$\therefore \bigcup A_i \in T.$

$\therefore T$ is topology on X.

7) let (X,T) T.S., let $A \subseteq X$, and $\forall p \in A$, $\exists G_p \in T$ such that $p \in G_p \subseteq A$ show that A is open set





sol: $\forall p \in A$

$\exists G_p \in T$

s.t. $p \in G_p \subseteq A$

$\therefore \bigcup \{G_p : p \in A\} = A$

but G_p is open $\forall p \in A$

اتحاد

so, A is a union of open sets, so A is open set.

شرط

by con. ③ of Topology. حسب شرط الثالث

الموضوع:

التاريخ: / /

12) let (X,T) T.s.
let $A \subseteq X$, $p \in X$.
when p is not limit point of A?
solution:-
case①:- $\exists G \in T$, $p \in G$

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$(G - \{p\}) \cap A = \emptyset$

Case②:- $\exists G \in T$, such that:
$p \in G$ and:

$G \cap A = \{p\}$

كل مجاميع جزئية من اكس هي مجموعة مفتوحة

13) let (X, T_d) be the discrete topology.

let $A \subseteq X$ find A'
solution:- A' derived set.

i.e. A' is the set of all limit points of A.

let $p \in X$.
is p limit point of A?
since $\{p\} \cap A = \emptyset$
or $\{p\} \cap A = \{p\}$

So, p is not limit point of p
$\therefore A' = \emptyset$

14) H.W. ص٢٥

15) let (X, T) T.S. and $A, B \subseteq X$. If $A \subseteq B$ Then

A' هي مجموعة نقاط لمت بوينت

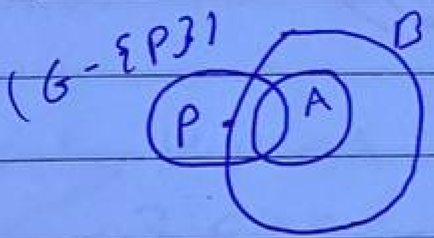
$A' \subseteq B'$ or every limit point of A is limite point of B

proof: let p limit point of A

i.e. $p \in A'$

i.e. $\forall\ G \in T$ s.t. $p \in G$ we have:

$(G - \{p\}) \cap A \neq \emptyset$



So, $(G - \{p\}) \cap B \neq \emptyset$

$\therefore p$ limit point of B

$p \in B'$

i.e. $A' \subseteq B'$

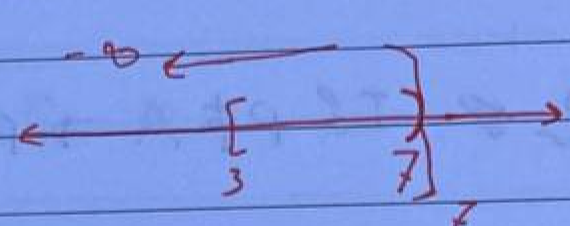
18) H.W.

20/ let $(\mathbb{R}, T_a)$ be a T.S. on The set of Real numbears.

$T_a = \{\mathbb{R}, \phi, \{E_a = (a, \infty)\}\}, \; a \in \mathbb{R}$

1) find the closed sets:

$T_a^c = \{\mathbb{R}, \phi, (-\infty, a]\}$



2) $cL([3, 7)) = ?$

$cL([3, 7)) = (-\infty, 7]$

3) $cL(\{4, 5, 22, 88\}) = ?$

$= (-\infty, 88]$

توضيح 21

21) (X, T_d)

$A^c \subseteq X$

$cL(A) = A$

$\therefore A^c \in T_d$

$\therefore A^c$ is open

تكملة 21) Determine The dense subset of X?

$\therefore A$ is closed

$\therefore cL(A) = A$

The only dense subset of X is X itself.

22/ H.W

التاريخ: / /

الموضوع: ______

23) let (X,T) T.s. , $A \subseteq X$, is closed iff $\forall p \in A'$, $p \in A$.

Proof:

$\Rightarrow$ let A closed subset of X.

T.P. $A' \subseteq A$?

i.e. T.P. If $p \notin A \Rightarrow p \notin A'$

let $p \notin A$

$\therefore p \in A^c$

but $A^c \in T$

* A^c is open , $A^c \in T$
* $p \in A^c$
* $A^c \cap A \in \emptyset$

$\therefore$ p is not limit point of A.

$\therefore p \notin A'$

$\boxed{\therefore A' \subseteq A}$

i.e. $\forall p \in A' \Rightarrow p \in A$

$\Leftarrow$ let $A' \subseteq A$

T.P. A is closed?

i.e. T.P. A^c is open

الموضوع: ____________ التاريخ: / /

i.e. T.P. $\forall p \in A^c$ is interior point of A.

so, let $p \in A^c$

$\therefore p \notin A'$ since:- $\boxed{p.\ A^c / A\ (A')}$

So, P is not limit point of A.

$\therefore \exists\ G \in T$ s.t. $p \in G$ $(G - \{p\}) \cap A = \emptyset$

but $p \notin A$.

$G \cap A = \emptyset$

$\therefore G \subseteq A^c$

$\therefore p \in G \subseteq A^c$

$\therefore$ P is interior point of A^c.

$\therefore A^c$ is open.

((Remark: A set B is open iff $\forall p \in B$, P is interior point of B)).

$(A^c)^c = A$ is closed.

30/ H.W./

Chapter two:-

bases and sub bases:-

اساسي

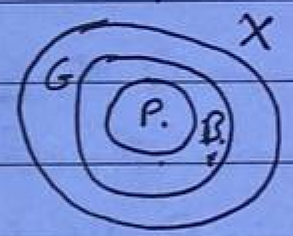
let (X, T) T.S., let $\beta \subseteq T$, β is a base of T on X

iff:-　　　　يقابل

*1) for every element of T can be represented as union of members of β.

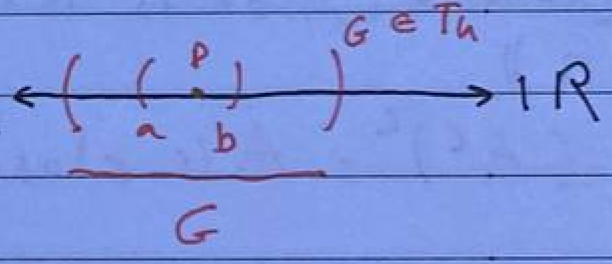
*equivalently:- $\forall G \in T$, $\forall p \in G$, $\exists B_i \in \beta$

such That: $p \in B_i \subseteq G$



Example:- let $(\mathbb{R}, T_u)$ be The usual topology on $\mathbb{R}$.

* $\beta = \{(a,b) : a, b \in \mathbb{R}\}$
  β is a base for T_u.



Example: (X, T_d) find The base of T_d on X.

Ex:- $X = \{a, b, c\}$
$T_d = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$
$\beta = \{\{a\}, \{b\}, \{c\}\}$

solution:- $\beta = \{\{P\} : P \in X\}$

To show That β is a base for T_d?

let $G \in T_d$
let $P \in G$
$P \in \underbrace{\{P\}}_{\in \beta} \subseteq G$.

مهم
Example: let $X = \{a, b, c\}$ show That :

$\beta = \{\{a,b\}, \{b,c\}\}$ is not base for any Topology T on X.

solution:-

T.P. β is not base on any T on X.

let β is a base for same topology T on X.

$\beta \subseteq T$

$\{a,b\} \in T$

$\{b,c\} \in T$

so $\{a,b\} \cap \{b,c\} = \{b\} \in T$

but if β base of T on X

so $\forall G \in T$ can represented (تمثل) as a Union of members of β.

but $G = \{b\} \in T$

$\{b\}$ can not be represented (تمثل) as a Union of members of β.

$\therefore \beta$ is not base of any T on X.

Definition:- let (X, T) T.s. , A collection $\Psi \subseteq T$

Ψ is subbase (اساس جزئي) for T on X iff:

$\Psi \to \cap \to \beta \to \cup \to T$

All The finite intersections of member of Ψ is a base β of T on X

Example: $X = \{a, b, c, d\}$

$\psi = \{\{a, b\}, \{b, c\}, \{d\}\}$ find The topology T on X generated by ψ. (Remark ψ is a subbase).

Solution: $\boxed{\psi \xrightarrow{\cap} \beta \xrightarrow{\cup} T}$

we should find all the finite intersections of elements of ψ to get β.

$\boxed{\psi \xrightarrow{\cap} \beta}$

$\beta = \{\{a, b\}, \{b, c\}, \{d\}, \{b\}, \emptyset, X\}$

$\beta \xrightarrow{\cup} T$

$T = \{\emptyset, X, \{a, b\}, \{b, c\}, \{d\}, \{b\}, \{a, b, c\}, \{a, b, d\}, \{b, d\}, \{b, c, d\}\}$

3) let (X, T) T.S., β be a base for T on X.

let β^* satisfied The following: $\beta \subseteq \beta^* \subseteq T$.

show that β^* is also base for T on X.

Proof: let $G \in T$

and β is a base for T

$G = \cup B_i$, $B_i \in \beta$

but $\beta \subseteq \beta^*$

التاريخ: / / الموضوع:

so $\beta_i \in \beta^*$, $\forall i$

$\therefore G = \bigcup \beta_i , \ \beta_i \in \beta^*$

$\therefore \beta^*$ is a base for T on X.

9/ H.W/

10/ find The topology T on The real line (مجموعة اعداد حقيقية) generated by the class A of sets:-

$\varphi = A = \{ B : B = [a, a+1] \}$. (فترة مغلقة)

solution:-

let $p \in \mathbb{R}$

Then $[p-1, p] \in A \subseteq T$

$[p, p+1] \in A \subseteq T$

$[p-1, p] \cap [p, p+1] = \{p\}$

$\{p\} \in T , \forall p \in \mathbb{R}$

so, The topology generated by A is the discrete Topology (T_d) on $\mathbb{R}$