CHAPER -5-

NUCLEAR MODELS

5.1 Constitution of the nucleus; Neutron-Proton hypothesis

We have seen that the atomic masses of different isotopes have values close to the integers representing their mass numbers. This is known as Aston's whole numbers rule, on the basis of which W.Prout, had advanced the hypothesis that the atoms of different elements were built up of different numbers of hydrogen atoms. However there are some inconsistencies in Prout's hypothesis. For example, elements like Chlorine and Copper have atomic weights considerably different from whole numbers (35.45 and 63.54 respectively). These observation forced abandonment of Prout's hypothesis. However, with the discovery of Aston's whole numbers rule about the isotopic masses, interest in Prout's hypothesis was revived.

The different isotopes have nearly integral atomic masses. Since the atomic mass of the isotope ^{1}H of hydrogen (1.007825) is nearly unity, it assume that the nucleus of the atom of an isotope of mass number A is made up of the nuclei of A numbers. Of hydrogen atoms of mass number 1 (^{1}H) which are nothing but protons, since each proton carries one electronic unit of positive charge (+e) this would make the total charge of A protons equal to +Ae. However, the charge of a nucleus is actually equal to +Ze, Z being the atomic number which is usually less than A. so obviously the atomic nucleus cannot be made up of A protons only. To get around the above difficulty, it was assumed that apart from the A protons, the nucleus contained (A-Z) electrons, each carrying the charge –e. This would make the total charge of the nucleus +Ze. Since the mass of the electron is much smaller than that of the proton, the total mass of the nucleus would still be close to the total mass of the A proton in it.

$M_N=Am_p+(A-Z)m_e$, where $m_e<<m_p$

$M_N=Am_p$, where $m_p\approx 1$ a.m.u

$M_N=A$

Nuclear Charge = +Ae-(A-Z)e

Nuclear Charge =Ae-Ae+Ze

Nuclear Charge =+Ze

This Proton-Electron hypothesis of the nuclear constitution has many flaws.

1- Electrons cannot remain within the nuclei which have radii of the order of 10^{-14} m or less. According to Heisenberg's uncertainty principle, the uncertainty in the momentum P of the electron in the nucleus would then be, $\Delta P=\hbar/\Delta X=\hbar/R\approx 10^{-34}/10^{-14}=10^{-20}$ Kg.m/s. An electron with momentum of this order of magnitude would have the energy,

$E \approx C\,\Delta P = 3x10^{8}x10^{-20}/1.6x10^{-13} \approx 20$ MeV. There is no experimental evidence of the existence of such high energy electrons within the nuclei of atoms. So the permanent existence of electrons within the nucleus is not possible they are actually created at the time of beta decay.

2- The angular momentum of the nuclei, the proton-electron hypothesis would pose serious difficulties. electron and proton are both spin 1/2 particles. Their total number in the nucleus should be A+A-Z=2A-Z. if this is even then the total spin of the nucleus (I) should be integral, while if it is odd, (I) should be half integral, depending on whether the total number of the particles is even or odd. An example if we take the ^{14}N nucleus (Z-7), the number of proton and electrons would be 2A-7=28-7=21 which is odd. So ^{14}N nucleus should have half-integral spin. But measurement gives (I=1) for this nucleus, which is contrary to expectation from the proton-electron hypothesis.

3- The magnetic moment of the nuclei
magnetic moment for electron $M_e=2S_e*e/2MC$, where $S_e=1/2\ \hbar$, $S_e=1/2$
$M_e=e\ \hbar/2M_eC = eh/4\pi M_eC = .99x10^{-23}$ j/Tesla. It is call Bohr Magneton.
Nuclear magnetic moment $M_N = 2I*e/2M_pC$, where(I) is the nuclear spin for nucleus
$M_N=2I*eh/4\pi M_pC=2I*0.5x10^{-26}$ j/Tesla, where $eh/4\pi M_pC$ is call nuclear Magneton . The intrinsic magnetic moment of the electron is about 1000 times larger than that of the nucleus. If the nucleus contains magnetic momentum. However, the measured values are much smaller, being of the order of the magnetic moments of the nucleons.

4- statistics of the nuclei, all elementary particles in nature can be grouped as fermions or bosons on the basis of the symmetry property of their wave-function. For particles of half-integral intrinsic spin (1/2, 3/2. --), the wave-function is anti symmetric. They obey Fermi-Dirac (F-D) statistics and are known a fermions. Electrons, protons and neutrons belong to this class. System containing N fermions will obey either F-D statistics or B-E statistics, depending on N is odd or even respectively. For example, the ^{14}N nucleus, since there would be 21 protons and electrons which is odd, the statistics obeyed should be F-D. However, experimental evidence shows that it obeys B-E statistics.

So all the above evidences show that the proton-electron hypothesis of the nuclear constitution cannot be correct.

In 1932 James Chadwick observed the emission of a neutral particle from the nucleus while performing an experiment on the artificial transmutation of elements. These particles were found to have the same mass as the protons, with spin is 1/2 and hence it is a fermions and were called neutrons. W. Heisenberg proposed that the nuclei were made up of protons and neutrons, a nucleus of mass number A and atomic number Z consists of Z protons and N=A-Z neutrons, so the total number of particles within the nucleus is equal to its mass number A. Since the mass of the proton and neutron are both nearly unity the mass of the atom will be close to its mass number which thus explains Aston's whole number rule. The nucleus will carry Z units of positive electronic charge.

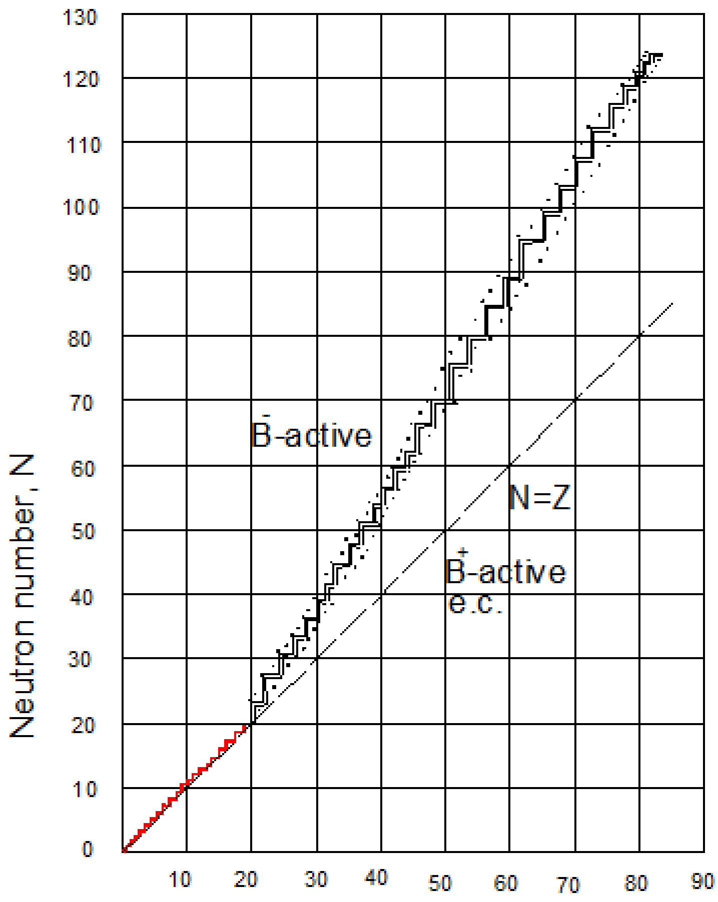
5.2 Nature of Nuclear Force the protons and neutrons are very strongly

The protons and neutrons are very strongly bound within the nucleus. The nature of the force which binds them together is basically different from the more familiar types of forces e.g. the Gravitational or the electromagnetic forces. The Gravitational force is far too weak to account for the nuclear binding. So far as the electromagnetic force is concerned, the neutrons being electrically neutral cannot have any electromagnetic interaction between themselves or with the protons. We must assume a type of force other than the above two to act between the nucleons within the nucleus. This force is very strongly attractive up to a certain maximum distance between the nucleons which is of the order of about 2 fm. This distance is known as the range of known as the strong interaction. A field theory of the inter nucleon force was first developed by the H Yukawa, who assumed the force to act through the inter charge of a particle of mass intermediate between that of an electron and a proton, just as the electromagnetic interaction acts through the intermediary of a virtual photon. This particle known as a pion. A pion is continually exchanged between two nucleons when they are at a distance less than about 2 fm. This type of force, known As the exchange force.

5.3 Inventory of Stable Nuclides

The naturally occurring elements can be grouped into two classes according to whether they have even or odd values of Z. Elements with even Z have larger number of stable isotopes than with odd Z. The latter have one or two stable isotopes. Fig. (5-1) shows the plot of the number of neutrons (N) against the number of protons Z for the stable nuclei of different elements. For the numbers of protons and neutrons are nearly equal to N/Z=1 for them and the stability line is equally inclined to the Z and N axes. For the heavier nuclei, the number of neutrons is higher than the number of protons so that N/Z becomes greater than 1, its highest value is about (1.6) for the very heavy nuclei. The plot of the nuclei in the N vs. Z graph is known as the Segre chart. The isotopes of all elements can be divided into four group: even Z even N (e-e), even Z odd N (o-o). the maximum number of stable isotopes is observed amongst the even-even group which is about 60% of the total number of stable isotopes. The even-odd and odd-even group are approximately equal in numbers and constitute about 20% each of the total. The numbers of stable odd-odd isotopes is the smallest only five. These are the isotopes ^{2}H (Z=1), ^{6}Li (Z=3), ^{10}B (Z=5), ^{14}N, (Z=7) and ^{159}Ta (Z=73).

The equality of Z and N for the lighter nuclei shows that the proton-proton and neutron-neutron forces are approximately equal within the nuclei. This is known as charge-symmetry of the nuclear force. In the heavier nuclei the coulomb repulsion between the protons tends to weaken the binding. To compensate for this the numbers of neutrons in them must be relatively higher, which increases the strength of binding. If the number of neutrons N is increased, keeping the number of protons constant then the nucleus formed goes to the left of the stability line. In such a nucleus, a neutron may spontaneously transform into a proton and the nucleus becomes β^- active. On the other hand, if the number of protons is increased keeping N constant the new nucleus formed falls on the right of the stability line. In such a nucleus, a proton may transform spontaneously into a neutron and the nucleus becomes β^+ active or electron-capture.



Fig(5-1) Neutron-proton diagram for stable nuclides

5.4 Nuclear Models

It may be noted that even if the exact nature of the inter nucleon interaction were known, it would have been extremely difficult to develop a satisfactory theory of the structure of the nucleus made up of a large number of neutrons and protons. Since it is impossible to solve the Schrodinger equation exactly for such a many body system. Various methods have been developed for tackling the problem with different degree of approximation. However, the problem is still far from being solved completely. Because of this difficulties in developing a satisfactory theory of nuclear structure, different models have been proposed for the nucleus, each of which can explain some of the different characteristics of the nucleus.

5.5 Liquid drop model

Each individual molecules within a liquid drop exerts an attractive force upon a group of molecules in its in immediate neighborhood**.** The force of interaction dose not extend to all the molecules within the drop. This is known as the saturation of the force. We saw before (Ch.2) that the binding energy E_B of a nucleus is proportional linearly to the number of nucleons within it, so that the binding fraction f_B (i.e. binding energy per nucleon) is nearly constant (≈8 MeV) for most nuclei. This fact shows a close resemblance of the nucleus with a liquid drop. Thus we come to the conclusion that the inter nucleon force within the nucleus attains a saturation value, so that each nucleon can interact only with a limited number of nucleons in its close vicinity. There are certain other points of resemblance between the nucleus of an atom and a liquid drop.

(i) The attractive force near the nuclear surface is similar to the force of surface tension on the surface of the liquid drop.

(ii) As in the case of a liquid drop, the density of the nuclear matter is independent of its volume.

(iii) Different types of particles, e.g. neutrons, protons, deuterons, α-particles etc. are emitted during nuclear reactions, these process are analogous to the emission of the molecules from the liquid drop during evaporation.

(iv) The internal energy of the nucleus is analogous to the heat energy within the liquid drop.

(v) The formation of a short lived compound nucleus by the absorption of a nuclear particle in a nucleus during a nuclear reaction is analogous to the process of condensation from the vapor to the liquid phase in the case of the liquid drop.

The liquid drop model is not very successful in describing the low lying excited states of the collective motion of the large number of nucleons involved, the model gives rise to closely spaced energy levels. Actually these are found to be quite widely spaced at low excitation energies.

Quick quiz 5.1 According to Proton-Electron hypothesis find the angular momentum and statistics of the ^{10}N (Z=5) nucleus.

Quick quiz 5.2 Define a virtual photon and a pion.

Quick quiz 5.3 localize the charge-symmetry of the nuclear force in segre chart.

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